

3.11 Sum & Difference Identities (Sine & Tangent)

Name _____ Date _____ Period _____

Find the exact values of the following sums or differences.

1. $\frac{\pi}{4} + \frac{\pi}{12}$

2. $\frac{3\pi}{4} - \frac{\pi}{3}$

3. $\frac{\pi}{6} - \frac{\pi}{4}$

Express each given angle as $\alpha + \beta$ or $\alpha - \beta$, where $\sin \alpha$ and $\sin \beta$ are known exactly.

4. 105°

5. -15°

6. -75°

Use appropriate identities to find the exact value of each expression. No calculators!

7. $\sin\left(\frac{7\pi}{12}\right)$

8. $\sin\left(\frac{5\pi}{12}\right)$

9. $\tan\left(\frac{-13\pi}{12}\right)$

10. $\tan\left(\frac{7\pi}{12}\right)$

11. $\sin(-15^\circ)$

12. $\tan(75^\circ)$

Simplify each expression by using appropriate identities. Do not use a calculator.

13. $\sin(23^\circ)\cos(67^\circ) + \cos(23^\circ)\sin(67^\circ)$

14. $\sin(2^\circ)\cos(7^\circ) - \cos(2^\circ)\sin(7^\circ)$

$$15. \quad \sin\left(-\frac{\pi}{2}\right)\cos\left(\frac{\pi}{5}\right) + \cos\left(\frac{\pi}{2}\right)\sin\left(-\frac{\pi}{5}\right)$$

$$16. \quad \frac{\tan\left(\frac{\pi}{9}\right) + \tan\left(\frac{\pi}{6}\right)}{1 - \tan\left(\frac{\pi}{9}\right)\tan\left(\frac{\pi}{6}\right)}$$

$$17. \quad \frac{\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{5}\right)}{1 + \tan\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{5}\right)}$$

Solve each problem. Write answers as reduced fractions.

$$18. \quad \text{Find the exact value of } \sin(\alpha + \beta) \text{ if } \sin \alpha = \frac{3}{5} \text{ and } \sin \beta = \frac{5}{13}, \text{ with } \alpha \text{ in quadrant II and } \beta \text{ in quadrant I.}$$

$$19. \quad \text{Find the exact value of } \sin(\alpha + \beta) \text{ if } \sin \alpha = \frac{7}{25} \text{ and } \sin \beta = \frac{-8}{17}, \text{ with } \alpha \text{ in quadrant II and } \beta \text{ in quadrant III.}$$

$$20. \quad \text{Find the exact value of } \sin(\alpha + \beta) \text{ if } \sin \alpha = \frac{2}{3} \text{ and } \sin \beta = \frac{-1}{2}, \text{ with } \alpha \text{ in quadrant I and } \beta \text{ in quadrant III.}$$

Write each expression as a function of α alone.

$$21. \quad \sin(\alpha - \pi) \sin(\alpha - \pi)$$

$$22. \quad \tan\left(\frac{\pi}{4} + \alpha\right)$$

Verify that each equation is an identity.

23. $\frac{\sin(x+y)}{\sin x \cos y} = 1 + \cot x \tan y$

24. $\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta$

25. $\sin(2x) = 2 \sin x \cos x$ Hint: $2x = x + x$

26. $\frac{\cos(\alpha - \beta)}{\sin(\alpha + \beta)} = \frac{1 + \tan \alpha \tan \beta}{\tan \alpha + \tan \beta}$

27. Complete the sum and difference identities.

a) $\cos(x + y) =$ _____

b) $\cos(x - y) =$ _____

28. One of the acute angles of a right triangle is 26° and its hypotenuse is 38.6 inches. Find the lengths of its legs to the nearest tenth of an inch.